Eur. Phys. J. C 1, 729–734 (1998)

THE EUROPEAN
PHYSICAL JOURNAL C

The three–form multiplet in $N = 2$ superspace

Maximilian F. Hasler[a]

Centre de Physique Théorique[b], C.N.R.S. Luminy, case 907, F–13288 Marseille, France (e–mail: hasler@cpt.univ-mrs.fr)

Received: 16 April 1997

Abstract. We present an $N = 2$ multiplet including a three–index antisymmetric tensor gauge potential, and describe it as a solution to the Bianchi identities for the associated fieldstrength superform, subject to some covariant constraints, in extended central charge superspace. We find that this solution is given in terms of an $8 + 8$ tensor multiplet subject to an additional constraint. We give the transformation laws for the multiplet as well as invariant superfield and component field lagrangians, and mention possible couplings to other multiplets. We also allude to the relevance of the 3–form geometry for generic invariant supergravity actions.

1 Introduction

Although $N = 2$ supersymmetric theories are not yet directly related to the observable world (at least in what concerns particle physics), they started playing an important role in modern theoretical physics in recent years. This is in particular due to the fact that they yield the first exactly solvable models in four dimensional field theories, in spite of the rich physics they display, including also non–perturbative phenomena [1].

In this paper, we discuss an $N = 2$ supermultiplet including a three–index antisymmetric tensor gauge potential. This multiplet will play a crucial role in various scenarios related to supersymmetry breaking, in analogy to the case in $N = 1$: For instance a gaugino condensate is associated, from a geometrical point of view, to the derivative $\mathrm{d}\mathcal{Q} = \mathrm{tr}\,\mathcal{F}^2$ of the Yang–Mills Chern–Simons form $\mathcal{Q} = \mathrm{tr}(\mathcal{A}\mathcal{F} - \frac{1}{3}\,\mathcal{A}^3)$, which is just a special case of a three–form potential. Similarily, curvature squared terms, which seem to be involved in another recently discussed supersymmetry breaking mechanism [2], are associated in the same way to the gravitational Chern–Simons forms. For $N = 1$, the three–form multiplet has been known for some time [3], and recently a rather complete description of its couplings to the $N = 1$ supergravity–matter system has been given [4].

However, to our knowledge, the corresponding $N = 2$ multiplet has not yet been constructed. It is the purpose of the present paper to fill this gap: First, we discuss the field content and the supersymmetry transformations on the level of component fields, and give an invariant lagrangian density for the multiplet. Then, we turn to a geometric description of the multiplet as fieldstrength of a three–form gauge potential in extended superspace. We find that this solution is given in terms of an $8 + 8$ tensor multiplet [5] ("linear superfield") subject to an additional constraint[1] similar to the case in $N = 1$. We also give the geometrical interpretation of the previously defined supersymmetry transformations. Finally, we give chiral superfield lagrangians, comment on the dynamics of this multiplet in various contexts, and mention possible couplings to $N = 2$ supergravity and to other multiplets. We also indicate the relevance of the three–form geometry for the construction of generic invariant actions.

Our spinor notations are those of Wess and Bagger [7], and concerning the internal structure group of $N = 2$ superspace, we adopted the conventions of [8]: In particular, we raise and lower internal SU(2)–indices from the left by the antisymmetric tensors ϵ_{BA} and ϵ^{BA} with $\epsilon^{12} = \epsilon_{21} = 1$.

2 The three–form multiplet

The set of fields that we will call the three–form multiplet Σ in the sequel consists of a three–index gauge potential $C_{\ell mn}(x)$, another two–index antisymmetric tensor $S_{mn}(x)$, an isotriplet of real scalar fields which we write as traceless, hermitean 2×2–matrix $Z^B{}_A(x)$, as well as an isodoublet of Weyl spinors $\zeta^A_\alpha(x)$, $\bar\zeta^{\dot\alpha}_A(x)$ and a real scalar auxiliary field $H(x)$:

$$\Sigma \;\sim\; \left(C_{\ell mn},\; S_{mn},\; Z^B{}_A;\; \zeta^A_\alpha,\; \bar\zeta^{\dot\alpha}_A \mid H\right) . \tag{1}$$

The field tensors

$$\widetilde{Z}^m = \frac{1}{2}\,\varepsilon^{mnk\ell}\partial_n S_{k\ell}\;, \qquad \widetilde{\Sigma} = -\partial_m \widetilde{C}^m \tag{2}$$

(where $C_{k\ell m} = \varepsilon_{k\ell mn}\widetilde{C}^n$ etc.) are invariant under the gauge transformations

[a] allocataire M. E. S. R.

[b] Unité Propre de Recherche 7061

[1] Another "extra–constrained" hypermultiplet seems to be known, but it is apparently impossible to write an action for this multiplet [6]

$$\delta S_{mn} = -\partial_m\beta_n + \partial_n\beta_m \qquad \delta\widetilde{C}^m = -\frac{1}{2}\,\varepsilon^{mnk\ell}\partial_n\gamma_{k\ell}\ . \tag{3}$$

We define the following supersymmetry transformations of spinorial parameters ξ_A^α and $\bar\xi^A_{\dot\alpha}$ for the multiplet:

$$\begin{aligned}
\delta S_{mn} &= 2\left(\xi_A\sigma_{mn}\zeta^A + \bar\xi^A\bar\sigma_{mn}\bar\zeta_A\right)\,,\\
\delta C_{\ell mn} &= \varepsilon_{\ell mnk}\left(\xi_A\sigma^k\bar\zeta^A + \bar\xi^A\bar\sigma^k\zeta_A\right)\,,\\
\delta Z^{BA} &= \sum^{BA}\left(\xi^B\zeta^A + \bar\xi^B\bar\zeta^A\right)\,,\\
\delta\zeta^A_\alpha &= \xi^A_\alpha\Big(i\,\partial^m\widetilde{C}_m - H\Big)\\
&\quad+\Big(i\,\partial^m Z^A{}_B - \delta^A_B\,\widetilde{Z}^m\Big)\left(\sigma_m\bar\xi\right)^B_\alpha\,,\\
\delta\bar\zeta^{\dot\alpha}_A &= \bar\xi^{\dot\alpha}_A\Big(i\,\partial^m\widetilde{C}_m + H\Big)\\
&\quad+\Big(i\,\partial^m Z^B{}_A + \delta^B_A\,\widetilde{Z}^m\Big)\left(\bar\sigma_m\xi\right)^{\dot\alpha}_B\,,\\
\delta H &= i\,\xi_A\partial\!\!\!/\bar\zeta^A - i\,\bar\xi^A\bar\partial\!\!\!/\zeta_A\ .
\end{aligned} \tag{4}$$

One can verify that these transformations close on the multiplet: The commutator $[\delta_2,\delta_1]$ of two supersymmetry transformations of parameters ξ_i yields a spacetime translation of parameter

$$\xi^m = 2i\left(\xi_{2A}\sigma^m\bar\xi_1^A + \bar\xi_2^A\bar\sigma^m\xi_{1A}\right) \tag{5}$$

on all fields of the multiplet, plus gauge transformations with field dependent parameters for the potentials, explicitely

$$\beta_m = \xi^n S_{nm} + 2i\left(\xi_{2A}\sigma_m\bar\xi_1^B + \bar\xi_2^B\bar\sigma_m\xi_{1A}\right)Z^A{}_B \tag{6}$$

and

$$\begin{aligned}
\gamma_{mn} &= \xi^\ell C_{\ell mn} + 2\left(\xi_{2A}\xi_1^A - \bar\xi_2^A\bar\xi_{1A}\right)S_{mn}\\
&\quad+4\left(\xi_{2A}\sigma_{mn}\xi_1^B - \bar\xi_2^B\bar\sigma_{mn}\bar\xi_{1A}\right)Z^A{}_B\ .
\end{aligned} \tag{7}$$

The geometric interpretation of these terms will be given later in this paper.

An invariant kinetic action for this multiplet is provided by the lagrangian density

$$\begin{aligned}
\mathcal{L}_{\rm kin} &= \tfrac{1}{2}\,H^2 + \tfrac{1}{2}\,(\partial_m\widetilde{C}^m)^2 + \tfrac{1}{2}\,\widetilde{Z}^m\widetilde{Z}_m\\
&\quad-\tfrac{1}{4}\,\partial^m Z^A{}_B\ \partial_m Z^B{}_A - \tfrac{i}{2}\,\zeta^A\ \overset{\leftrightarrow}{\partial\!\!\!/}\ \bar\zeta_A\ .
\end{aligned} \tag{8}$$

It should be noted here that $C_{\ell mn}$ does not propagate physical degrees of freedom. On–shell, the above action describes the same 4+4 physical states as the usual tensor multiplet [5], the 4 physical spin–0 degrees of freedom being given by the triplet of scalars and the antisymmetric tensor.

Thus, in analogy to the case of the $N=1$ three–form multiplet [3], the pseudoscalar auxiliary field of the usual matter multiplet is replaced by the fieldstrength of the three–form potential.

Actually, the multiplet can be decomposed into one $N=1$ three–form multiplet and one $N=1$ antisymmetric tensor multiplet [9]: This is done by writing

$$Z^B{}_A \sim \begin{pmatrix} L & \overline{T}\\ T & -L\end{pmatrix}\,, \qquad L = L^\dagger \tag{9}$$

and splitting up the $N=2$ multiplet into the two $N=1$ multiplets

$$\begin{aligned}
\varSigma\ \rightsquigarrow\ &(\ L,\ S_{mn};\ \zeta^1_\alpha,\ \bar\zeta_1^{\dot\alpha}\ )\\
&+\ (\ T,\ \overline{T},\ C_{\ell mn};\ \zeta^2_\alpha,\ \bar\zeta_2^{\dot\alpha}\ |\ H\ )\ .
\end{aligned} \tag{10}$$

Then, the supersymmetry transformations of parameter $\xi^1_\alpha,\ \bar\xi_1^{\dot\alpha}$ correspond to the usual $N=1$ transformations of these two multiplets, while the parameters $\xi^2_\alpha,\ \bar\xi_2^{\dot\alpha}$ mix them in a nontrivial way. We shall reconsider this issue on the level of superfields after the discussion of the superspace Bianchi identities in the following section.

3 Superspace geometry and Bianchi identities

We find that the previously introduced multiplet can be described as the components of the fieldstrength tensor $\varSigma = \mathrm{d}C$ of a three–form potential in extended $N=2$ superspace. We include an additional bosonic coordinate $z,\ \bar z$ which allows for the description of a supersymmetry algebra including a central charge [10]. Thus, we consider torsion $T^{\mathcal{A}} = \mathrm{d}E^{\mathcal{A}}$ ($\mathcal{A}\sim a,\ {}^\alpha_A,\ {}^A_{\dot\alpha},\ z,\ \bar z$) with the following nonzero elements:

$$T_{\gamma\,\beta}^{CB\ z} = 2i\,\epsilon^{CB}\,\epsilon_{\gamma\beta}\ , \qquad T^{\dot\gamma\,\dot\beta\ \bar z}_{CB} = 2i\,\epsilon_{CB}\,\epsilon^{\dot\gamma\dot\beta}\ , \tag{11}$$

and, as usual,

$$T_{\gamma\,B}^{C\,\dot\beta\ a} = 2i\,\delta^C_B\,(\sigma^a)_\gamma{}^{\dot\beta}\ . \tag{12}$$

By Poincaré's lemma dd $=0$, they define the commutators

$$\big(\mathcal{D}_{\mathcal{C}},\mathcal{D}_{\mathcal{B}}\big) = -T_{\mathcal{CB}}{}^{\mathcal{A}}\mathcal{D}_{\mathcal{A}}\ . \tag{13}$$

However, the superfields of the multiplet described here, as well as the gauge parameters, are taken independent of $z,\bar z$,

$$\partial_z C_{\mathcal{CBA}} = 0\ , \qquad \partial_{\bar z} C_{\mathcal{CBA}} = 0\ , \tag{14}$$

i.e., the central charge is acting trivially on the multiplet. Furthermore, we impose the covariant constraints

$$\varSigma_{\underline{\delta}\underline{\gamma}\underline{\beta}\mathcal{A}} = 0\ , \qquad \varSigma_{\underline{\delta}\underline{\gamma}\,z\bar z} = 0\ , \qquad \Big(\ \underline{\delta}\sim {}^D_\delta,\ {}^{\dot\delta}_D\ \Big) \tag{15}$$

and

$$\varSigma_{z\,c\,\beta\,A}^{\ \ \ \ B\,\dot\alpha} = 2\,\sigma_{c\,\beta}{}^{\dot\alpha}\,Z^B{}_A\ , \qquad \varSigma_{\bar z\,c\,\beta\,A}^{\ \ \ \ B\,\dot\alpha} = -2\,\sigma_{c\,\beta}{}^{\dot\alpha}\,Z^B{}_A \tag{16}$$

on the fieldstrength[2].

[2] The latter constraint should be considered as reality condition on $Z^B{}_A$. An antihermitean part of $Z^B{}_A$ would just enlarge the field content of the multiplet by another 8+8 off–shell degrees of freedom

Then the Bianchi identities $\mathrm{d}\Sigma=0$, or

$$E^{\mathcal{A}}E^{\mathcal{B}}E^{\mathcal{C}}E^{\mathcal{D}}E^{\mathcal{E}}\left(\mathcal{D}_{\mathcal{E}}\Sigma_{\mathcal{DCBA}}+2\,T_{\mathcal{ED}}{}^{\mathcal{F}}\,\Sigma_{\mathcal{FCBA}}\right)=0\ , \quad (17)$$

give all other elements of $\Sigma_{\mathcal{DCBA}}$ in terms of $Z^B{}_A$ and its derivatives: First, the identities with indices $z\bar{z}$ imply

$$\Sigma_{z\bar{z}\,\mathcal{BA}}=0\ . \quad (18)$$

(These identities are just the Bianchi identities for a Yang–Mills fieldstrength $F_{\mathcal{BA}}$ in ordinary $N=2$ superspace without the z–coordinates [11], with the Yang–Mills superfields occuring in $F_{\underline{\beta\alpha}}$ set to zero.)
Next, the identities with one index z correspond to the Bianchi identities $\mathrm{d}Z=0$ for

$$Z_{\mathcal{CBA}}\equiv i\Sigma_{z\,\mathcal{CBA}} \quad (19)$$

such that $Z=\mathrm{d}S$ for some 2–form S, which is readily identified[3] as

$$S_{\mathcal{BA}}=-i\,C_{z\,\mathcal{BA}}\ . \quad (20)$$

These identities imply [12]

$$Z^A{}_A=0\ ,\qquad \mathcal{D}_\gamma^{(C}Z^{BA)}=0=\overline{\mathcal{D}}^{\dot\gamma}_{(C}Z_{BA)}\ , \quad (21)$$

and give

$$Z_{cb\,\alpha}^{\ \ A}=2\,(\sigma_{cb}\zeta)_\alpha^A\ ,\qquad Z_{cb\,A}^{\ \ \dot\alpha}=2\,(\bar\sigma_{cb}\bar\zeta)_A^{\dot\alpha}\ , \quad (22)$$

with

$$\zeta_\alpha^A=\tfrac{1}{3}\,\mathcal{D}_\alpha^B Z^A{}_B\ ,\qquad \bar\zeta_A^{\dot\alpha}=\tfrac{1}{3}\,\overline{\mathcal{D}}_B^{\dot\alpha}Z^B{}_A\ , \quad (23)$$

and finally

$$Z_{cba}=\tfrac{1}{8}\,\varepsilon_{dcba}\,(\sigma^d)^\alpha{}_{\dot\alpha}\left(\mathcal{D}_\alpha^A\bar\zeta_A^{\dot\alpha}-\overline{\mathcal{D}}_A^{\dot\alpha}\zeta_\alpha^A\right)\ . \quad (24)$$

As usual, the "reduced identities" (21) already imply that $\varepsilon^{dcba}\partial_d Z_{cba}=0$.
Then, the other components of $\mathrm{d}\Sigma=0$ determine the components of Σ without $z,\bar z$ indices to be

$$\begin{aligned}\Sigma_{dc\,\beta\,\alpha}^{\ \ BA}&=-4\,(\sigma_{dc})_{\beta\alpha}Z^{BA}\ ,\\ \Sigma_{dc\,BA}^{\ \ \dot\beta\,\dot\alpha}&=4\,(\bar\sigma_{dc})^{\dot\beta\dot\alpha}Z_{BA}\ ,\end{aligned} \quad (25)$$

and

$$\begin{aligned}\Sigma_{dcb\,\alpha}^{\ \ \ A}&=-\varepsilon_{dcba}\,(\sigma^a\bar\zeta)_\alpha^A\ ,\\ \Sigma_{dcb\,A}^{\ \ \ \dot\alpha}&=-\varepsilon_{dcba}\,(\bar\sigma^a\zeta)_A^{\dot\alpha}\ ,\end{aligned} \quad (26)$$

where we chose to adopt the additional constraint $\Sigma_{dc\,\beta\,A}^{\ \ B\,\dot\alpha}=0$ which turns out to be conventional, i.e. just corresponds to a shift of the potential $C_{\ell mn}$ by the trace of this fieldstrength component. Finally, the vectorial fieldstrength is given by

$$\begin{aligned}\Sigma_{dcba}&=\varepsilon_{dcba}\widetilde{\Sigma}\ ,\\ \widetilde{\Sigma}&=\frac{i}{8}\left(\mathcal{D}_A^\alpha\zeta_\alpha^A+\mathcal{D}_{\dot\alpha}^A\bar\zeta_A^{\dot\alpha}\right)\ .\end{aligned} \quad (27)$$

On the other hand, one infers of course from the explicit definition of $\Sigma=\mathrm{d}C$ that

$$\widetilde{\Sigma}=-\tfrac{1}{6}\,\varepsilon^{dcba}\,\partial_d C_{cba}\ , \quad (28)$$

therefore the above result should be seen as an additional constraint on the superfield $Z^B{}_A$, requiring the imaginary part of its highest component to be a total derivative,

$$\left(\mathcal{D}_B^\alpha\mathcal{D}_\alpha^A+\overline{\mathcal{D}}_{\dot\alpha}^A\overline{\mathcal{D}}_B^{\dot\alpha}\right)Z^B{}_A=\partial_d\left(4i\,\varepsilon^{dcba}C_{cba}\right)\ . \quad (29)$$

This concludes the analysis of the Bianchi identities; no other restrictions on $Z^B{}_A$ are found. Note that we actually used here the same symbols for the superfields than for their lowest components which are precisely the fields presented in the preceding section. For convenience, we summarize their definitions here once again, denoting the projection on $\theta=0$ as usual by a vertical bar:

$$Z^B{}_A(x)=Z^B{}_A\,|\ ,\qquad S_{mn}(x)=-i\,C_{z\,mn}\,|\ , \quad (30)$$

$$\zeta_\alpha^A(x)=\frac{1}{3}\,\mathcal{D}_\alpha^B Z^A{}_B\,|\ ,\qquad \bar\zeta_A^{\dot\alpha}(x)=\frac{1}{3}\,\overline{\mathcal{D}}_B^{\dot\alpha}Z^B{}_A\,|\ , \quad (31)$$

$$\begin{aligned}\widetilde{Z}^m(x)&=\frac{1}{2}\,\varepsilon^{mnk\ell}\,\partial_n S_{k\ell}(x)\\ &=\frac{1}{24}\,(\sigma^m)^\alpha{}_{\dot\alpha}\left(\overline{\mathcal{D}}_B^{\dot\alpha}\mathcal{D}_\alpha^A-\mathcal{D}_\alpha^A\overline{\mathcal{D}}_B^{\dot\alpha}\right)Z^B{}_A\,|\ ,\end{aligned} \quad (32)$$

$$\begin{aligned}\widetilde{\Sigma}(x)&=-\frac{1}{6}\,\varepsilon^{k\ell mn}\,\partial_k C_{\ell mn}(x)\\ &=\frac{i}{24}\left(\mathcal{D}_B^\alpha\mathcal{D}_\alpha^A+\overline{\mathcal{D}}_{\dot\alpha}^A\overline{\mathcal{D}}_B^{\dot\alpha}\right)Z^B{}_A\,|\ ,\end{aligned} \quad (33)$$

and there remains just one real scalar auxiliary field, defined as

$$H(x)=\frac{1}{24}\left(\overline{\mathcal{D}}_{\dot\alpha}^A\overline{\mathcal{D}}_B^{\dot\alpha}-\mathcal{D}_B^\alpha\mathcal{D}_\alpha^A\right)Z^B{}_A\,|\ . \quad (34)$$

Observe that the additional constraint (29) is analogous to the case in $N=1$: There, the three–form multiplet also corresponds to (anti–)chiral superfields

$$\bar D^{\dot\alpha}T=0\ ,\qquad D_\alpha\overline{T}=0\ , \quad (35)$$

satisfying the additional constraint

$$D^\alpha D_\alpha T-\bar D_{\dot\alpha}\bar D^{\dot\alpha}\overline{T}\propto\varepsilon^{dcba}\partial_d C_{cba}\ . \quad (36)$$

Considering the decomposition (9) of $Z^B{}_A$, we find the chirality constraints (35) on T from the constraints (21), $\mathcal{D}_\alpha^{(C}Z^{BA)}=0$, by setting all indices to one, while the

[3] To be precise, from the reality condition (16) follows that $\Sigma_{z\,\mathcal{CBA}}=-\Sigma_{\bar z\,\mathcal{CBA}}$, such that $S'_{\mathcal{BA}}=i\,C_{\bar z\,\mathcal{BA}}$ has the same fieldstrength Z and therefore describes the same physical object. (Roughly speaking, $S'_{\mathcal{BA}}$ and $S_{\mathcal{BA}}$ differ at most by a gauge transformation.)

linearity constraints on L and the additional constraint on T, $\overline{T}$ are most easily recovered from the relations

$$\begin{aligned} \mathcal{D}^B_\beta \zeta^A_\alpha &= -\epsilon^{BA}\,\epsilon_{\beta\alpha}\,(H + i\,\widetilde{\Sigma}) \\ \overline{\mathcal{D}}^{\dot\beta}_B \bar\zeta^{\dot\alpha}_A &= \epsilon_{BA}\,\epsilon^{\dot\beta\dot\alpha}\,(H - i\,\widetilde{\Sigma})\ , \end{aligned} \tag{37}$$

again by setting the internal index B of the spinorial derivative to one.

To conclude this section, we now turn back to the previously introduced supersymmetry transformations. We define them to be superspace diffeomorphisms of parameter $\xi^{\mathcal{A}}$ such that the vielbeins $E^{\mathcal{A}}$ remain invariant, i.e.,

$$\begin{aligned} &\mathcal{L}_\xi E^{\mathcal{A}} \equiv (\iota_\xi + \mathrm{d})^2 E^{\mathcal{A}} = 0 \\ &\Longleftrightarrow\ \mathcal{D}_{\mathcal{B}}\xi^{\mathcal{A}} = -\xi^{\mathcal{C}}T_{\mathcal{CB}}{}^{\mathcal{A}}\ . \end{aligned} \tag{38}$$

This restricts the parameters to be x^m and $z, \bar z$–independent, and the θ and $\bar\theta$–components of ξ^a and $\xi^{\underline{z}}$ to be given in terms of ξ^α_A and $\bar\xi^A_{\dot\alpha}$. We combine these diffeomorphisms with a field dependent gauge transformation

$$\delta_\gamma C = -\mathrm{d}\gamma \qquad \text{with} \qquad \gamma = \iota_\xi C \tag{39}$$

in order to obtain the very simple transformation laws

$$\delta C = \iota_\xi \Sigma\ , \qquad \delta\Sigma_{\mathcal{DCBA}} = \iota_\xi \mathrm{d}\,\Sigma_{\mathcal{DCBA}}\ . \tag{40}$$

The relevant components of these formulae, or rather their values for $\theta = 0$, were given in the previous section. (We take the lowest component of ξ^m and $\xi^{\underline{z}}$ to vanish for the supersymmetry transformations; this is of course consistent with the condition (38).)

One can show that the commutator $[\delta_2, \delta_1]$ of two such transformations yields again a transformation of the same type, but of parameter

$$\xi^{\mathcal{A}} = -\iota_{\xi_2}\iota_{\xi_1}T^{\mathcal{A}} \equiv -\xi_2^{\mathcal{B}}\,\xi_1^{\mathcal{C}}\,T_{\mathcal{CB}}{}^{\mathcal{A}} \tag{41}$$

plus a gauge transformation of parameter

$$\gamma = -\iota_{\xi_2}\iota_{\xi_1}\Sigma\ , \qquad (\gamma)_{\mathcal{BA}} = -\xi_2^{\mathcal{C}}\,\xi_1^{\mathcal{D}}\,\Sigma_{\mathcal{DCBA}}\ . \tag{42}$$

Disentangeling furtherly, we can describe this combined transformation also as a plain diffeomorphism of parameter ξ plus a gauge transformation of parameter $\gamma + \iota_\xi C$, which then yields the formulae given in the first section.

We also wish to comment on the piece in these transformations that stems from the $z, \bar z$–component of the parameter ξ: It actually has the form of something one could view as a kind of "relic" central charge transformation, acting nontrivially only on the three–form potential, which becomes shifted by the antisymmetric tensor's fieldstrength,

$$\delta_z C_{cba} = \Delta \cdot Z_{cba}\ , \qquad \Delta = -i\left(\xi^z - \bar\xi^{\bar z}\right)\ . \tag{43}$$

However, according to the definition of $Z = \mathrm{d}S$, this also is nothing else than a gauge transformation, namely of parameter $\gamma = \Delta \cdot S$.

4 Superfield lagrangians

The previously given kinetic action for the three–form multiplet can actually be obtained as the θ^4 component of a chiral superfield lagrangian as follows: First, we consider an explicit solution to the constraints on Σ, namely $C_{z\mathcal{BA}} = i\,S_{\mathcal{BA}}$, $C_{\bar z\mathcal{BA}} = -i\,S_{\mathcal{BA}}$, with

$$\begin{aligned} &S^{BA}_{\beta\,\alpha} = 4\,\epsilon^{BA}\,\epsilon_{\beta\alpha}\overline{S}\ , \qquad S^{\dot\beta\,\dot\alpha}_{BA} = -4\,\epsilon_{BA}\,\epsilon^{\dot\beta\dot\alpha}\,S \\ &S^{B\,\dot\alpha}_{\beta\,A} = 0\ . \end{aligned} \tag{44}$$

Here, S and $\overline{S}$ are (anti–)chiral superfields, in terms of which the fieldstrength multiplet is then given by

$$Z^B{}_A = \frac{1}{4}\left(\mathcal{D}^\varphi_A\mathcal{D}^B_\varphi S - \overline{\mathcal{D}}^B_{\dot\varphi}\overline{\mathcal{D}}^{\dot\varphi}_A\overline{S}\right)\ . \tag{45}$$

(Note that S and $\overline{S}$ must derive from the same real prepotential, such that the imaginary part of their highest component is indeed a total derivative $\sim \partial_m\widetilde{C}^m$.) Then, the previously given kinetic action can be written as

$$\mathcal{L}_{\mathrm{kin}} = \Re\!e \int \mathrm{d}^4\theta\, SZ\ , \qquad Z = -\frac{1}{8}\,\overline{\mathcal{D}}^A_{\dot\varphi}\overline{\mathcal{D}}^{\dot\varphi}_B Z^B{}_A\ . \tag{46}$$

(Here the chiral volume element is normalized such that $\int \mathrm{d}^4\theta\ \ \theta^4 = 1$.) Due to the constraints (21) on $Z^B{}_A$, this superfield Z is actually a vector superfield, and the above lagrangian is indeed invariant under the gauge transformations of S [12].

We also want to mention that, given this explicit solution, one can add a mass term

$$\mathcal{L}_{\mathrm{mass}} = m^2 \int \mathrm{d}^4\theta\, S^2 + h.c. \tag{47}$$

which breaks the gauge invariance. (For the usual tensor multiplet, this lagrangian has first been considered in [5]). This yields a theory for a massive multiplet with twice the number of physical fields: $\mathcal{L}_{\mathrm{mass}}$ includes an explicit mass term for a Dirac spinor made of ζ^A_α and $m\mathcal{D}^A_\alpha S|$, and for $Z^B{}_A$ and the antisymmetric tensor S_{mn}. Moreover, an additional triplet of bosons, $X^B{}_A \sim m(\mathcal{D}^\varphi_A\mathcal{D}^B_\varphi S + \overline{\mathcal{D}}^B_{\dot\varphi}\overline{\mathcal{D}}^{\dot\varphi}_A\overline{S})$, appears as auxiliary fields. Finally, the diagonalization of the fields H and $\widetilde{\Sigma}$ yields a mass term for mS. However, this is a slightly delicate point: In complete analogy with the case in $N = 1$, one cannot "eliminate" $\widetilde{\Sigma} = -\partial^m C_m$ directly, as it is not really an auxiliary field. Moreover, its equation of motion only requires $\widetilde{\Sigma}$ to be a constant, but not to vanish. As one can see from the supersymmetry transformations, a nonvanishing constant would give rise to an inhomogeneous transformation law of the spinor field, which seems to indicate broken supersymmetry. For a more detailed discussion of these issues, see for example [13].

We wish to mention that the above lagrangians, and even a much larger class of N=2 invariant actions, find a nice geometric interpretation within the 3–form geometry. Actually, for an arbitrary superfield $L^B{}_A$ satisfying

$$D^{(A}_\alpha L^{BC)} = 0\ , \qquad (\partial_z \mp \partial_{\bar z})L^B{}_A = 0\ , \tag{48}$$

we find that

$$\mathcal{L} = \frac{-1}{24}\big(D^\alpha_A D^B_\alpha \mp \bar{D}^B_{\dot\alpha}\bar{D}^{\dot\alpha}_A\big)L^A{}_B \tag{49}$$

leads to an invariant action; more precisely

$$\delta\mathcal{L} = -\tfrac{i}{3}\partial_m\big(\bar\xi^B\bar\sigma^m D_A \mp \xi_A\sigma^m\bar{D}^B\big)\big(L^A{}_B - \tfrac{1}{2}\,\delta^A_B\,L^F{}_F\big)\ . \tag{50}$$

The formula (49) applies not only to the case of z–independent chiral lagrangians ($L^A{}_B = D^A D_B X \mp \bar{D}^A\bar{D}_B\overline{X}$ with $\bar{D}^{\dot\alpha}_A X = 0$), but can be used to obtain the lagrangians for the Fayet–Sohnius hypermultiplet [14,10] with $L^A{}_B = \bar\phi^A(m - \frac{i}{2}\overset{\leftrightarrow}{\partial})\phi_B$ and for the vector–tensor multiplet [15] with $L^A{}_B = W^A W_B - \overline{W}^A\overline{W}_B$.

The invariance of the lagrangian (49) finds a natural explication in the 3–form geometry: In fact, consider a 4–form $L = \frac{1}{4!}\,E^{\mathcal{A}}E^{\mathcal{B}}E^{\mathcal{C}}E^{\mathcal{D}}L_{\mathcal{DCBA}}$ which verifies $\mathrm{d}L = 0$. Then, a supergravity transformation gives

$$\delta L \equiv (\iota_\xi + d)^2 L = \mathrm{d}\iota_\xi L\ , \tag{51}$$

i.e., L transforms into a total derivative. Projecting on the spacetime components, one finds thus that the lagrangian

$$\mathcal{L} = e\,{}^*L \equiv \frac{e}{24}\varepsilon^{k\ell mn}L_{k\ell mn}(x) \tag{52}$$

(with $e = \det e_m{}^a$) provides an invariant action, since it transforms into

$$\delta\mathcal{L} = \partial_m\Big(\frac{e}{6}\,\varepsilon^{k\ell mn}\xi^{\underline{\alpha}}L_{\underline{\alpha}k\ell n}\Big)\ . \tag{53}$$

As we showed, the solution of $\mathrm{d}L = 0$ subject to constraints (15) gives all components of L in terms of $L_{\underline{z}\,c\,\beta A}{}^{B\dot\alpha} = 2i\,\sigma_{c\beta}{}^{\dot\alpha}L_{\underline{z}}{}^B{}_A$ and its derivatives. (Eq. (52) reduces in the flat case to eqs. (49) resp. (33) for appropriate choices of $L_{\underline{z}}{}^B{}_A$). This provides therefore a generic method to construct invariant lagrangians (52) from arbitrary "linear superfields" $L_{\underline{z}}{}^B{}_A$. (An similar remark in the case of $N=1$ supergravity lagrangians has been made in [16].)
In view of the large amount of necessary definitions and technicalities, we leave the details of the calculations in $N=2$ supergravity and a more elaborate discussion of this issue to a separate publication.

5 Couplings to other multiplets

As the 3–form gauge multiplet is a special case of a tensor multiplet, it allows for self–couplings and couplings to other multiplets in the same way than they do. Notably one could consider the analogue of the action for the improved tensor multiplet [17], which can be coupled to a general $N=2$ supergravity background [18]. For this multiplet, one also can add a mass term involving only the invariant fieldstrength multiplet. The massive three–form multiplet should actually be of interest mainly in the supergravity–coupled case.

On the other hand, another subject of special interest related to the three–form multiplet are of course couplings to other gauge multiplets that can be formulated on geometric grounds. In analogy to $N=1$ [19,13] we consider, for example, a modified field tensor

$$\varSigma = \mathrm{d}C + \kappa\,H\wedge A \tag{54}$$

where $H = \mathrm{d}B$ is the fieldstrength of a two–form potential and A is a U(1) gauge potential. This gives rise to the modified Bianchi Identity

$$\mathrm{d}\varSigma = \kappa\,H\wedge F\ ,\qquad F = \mathrm{d}A\ , \tag{55}$$

which yields in particular a modified fieldstrength $\widetilde{\varSigma}$, including the spinorial superpartners of A_m and B_{mn}. This leads to dynamics similar to the case in $N=1$, with notably quartic potential terms for the spinor fields, which might again be of interest in the scenarios alluded to in the introduction. These topics are still to be worked out in detail and will be discussed elsewhere.

Acknowledgements. I should like to thank R. Grimm for introducing me to supersymmetry and for useful suggestions during the course of this work.

References

1. N. Seiberg, E. Witten. Electric–magnetic duality, monopole condensation, and confinement in $N=2$ supersymmetric Yang–Mills theory. Nucl. Phys. B426 (1994), 19–52. Erratum: Nucl. Phys. B430 (1994), 485–486. (hep-th/9407087).
2. A. Hindawi, B. A. Ovrut, D. Waldram. Higher Derivative Gravitation and a New Mechanism For Supersymmetry Breaking in Four Dimensions. UPR-685-T (hep-th/9509054), September 1995
3. S. J. Gates Jr. Super p–form gauge superfields. Nucl. Phys. B184 (1981), 381–390
4. P. Binétruy, F. Pillon, G. Girardi, R. Grimm. The 3–form multiplet in supergravity. Nucl. Phys. B477 (1996), 175–202. (hep-th/9603181)
5. J. Wess. Supersymmetry and internal symmetry. Acta Physica Austriaca 41 (1975), 409–414
6. M. G. Faux. Private communication, June 1996
7. J. Wess, J. Bagger. Supersymmetry and Supergravity. Princeton Series in Physics. Princeton University Press, Princeton, 1983. 2nd edition 1992
8. M. Müller. Minimal $N=2$ supergravity in superspace. Nucl. Phys. B282 (1987), 329–348
9. S. Ferrara, B. Zumino, J. Wess. Supergauge multiplets and superfields. Phys. Lett. 51B (1974), 239–241
10. M. F. Sohnius. Supersymmetry and central charges. Nucl. Phys. B138 (1978), 109–121
11. R. Grimm, M. Sohnius, J. Wess. Extended supersymmetry and gauge theories. Nucl. Phys. B133 (1978), 275–284
12. M. Müller. Chiral actions for minimal $N=2$ supergravity. Nucl. Phys. B289 (1987), 557–572
13. M. F. Hasler. Geometrische Methoden und unkonventionelle Multipletts in supersymmetrischen Theorien. Diplomarbeit, Universität Karlsruhe, March 1996

14. P. Fayet. Fermi–Bose hypersymmetry. Nucl. Phys. B113 (1976), 135–155
15. A. Hindawi, B. A. Ovrut, D. Waldram. Vector–tensor multiplet in N=2 superspace with central charge. Phys. Lett. B392 (1997), 85–92 (hep-th/9609016).
16. G. Girardi, R. Grimm. $N = 1$ supergravity. Topological classes and superspace geometry in four dimensions. Phys. Lett. B260 (1991), 365–371
17. B. de Wit, M. Roček. Improved tensor multiplets. Phys. Lett. 109B (1982), 439
18. B. de Wit, R. Philippe, A. V. Proeyen. The improved tensor multiplet in $N = 2$ supergravity. Nucl. Phys. B219 (1983), 143
19. R. Grimm. Supergravity, linear multiplets, and Chern–Simons forms. In: Proc. U.S.–Polish Workshop on Physics from Planck Scale to Electroweak Scale (Warsaw, September 1994), eds. P. Nath, T. Taylor, and S. Pokorski (World Scientific, Singapore, 1995), pp. 277–286. (hep-th/9510073)